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DEFORMATION OF AN ELASTIC ANISOTROPIC MICRO-INHOMOGENEOUS HALF-SPACE*

V.V. PODALKOV and V.A. ROMANOV

There is considered the problem of deformation of a transversally-isotropic micro-inhomogeneous half-space under macro-homogeneous stress-strain state conditions. It is assumed that the half-space boundary is parallel to the plane of isotropy of the medium, filling it, and the elastic parameters governing the medium are independent of the coordinate normal to the boundary, and are random functions of two coordinates. The problem is solved by the method of perturbations. Dispersions of the deformation are constructed by this method in a first approximation. The behavior of the deformation dispersion is investigated as a function of the coordinate normal to the plane of isotropy.

1. In a transversally-isotropic inhomogeneous half-space $x_3 \geqslant 0$ let a macro-homogeneous state of stress and strain be realized

$$\sigma_{ij}^{(0)} = \langle \sigma_{ij} \rangle, \quad \varepsilon_{ij}^{(0)} = \langle \varepsilon_{ij} \rangle \tag{1.1}$$

Here and henceforth, the angular brackets will denote the operation of mathematical expectation.

Hooke's law for a transversally-isotropic medium has the form /1/

$$\begin{aligned} \sigma_{ii} &= (\lambda + \varkappa\delta_{i3})\,\theta + 2\mu\varepsilon_{ii} + (\varkappa + \gamma + 4\rho)\,\varepsilon_{33}\delta_{i3} \\ \sigma_{ij} &= 2\mu\varepsilon_{ij} + 2\rho\varepsilon_{ij}\,(\delta_{j3} + \delta_{i3}) \end{aligned} \tag{1.2}$$

Here $i \neq j;\ i, j = 1, 2, 3$, and there is no summation over i, $\lambda,\ \mu,\ \varkappa,\ \gamma,\ \rho$ are elastic moduli of the transversally-isotropic medium, $\theta = \varepsilon_{kk}$, and δ_{ij} is the Kronecker delta.

Let us represent the elastic characteristics of the material $q(\mathbf{x}_*)$ as well as the displacement of the deformations and stresses $A(\mathbf{x})$ in the form /2/

$$q(\mathbf{x}_*) = q^{(0)} + q^{(1)}(\mathbf{x}_*), \quad q^{(0)} = \langle q(\mathbf{x}_*) \rangle \tag{1.3}$$

$$\mathbf{x}_* = \{x_1;\ x_2\}, \quad q = \lambda,\ \mu,\ \varkappa,\ \gamma,\ \rho$$

$$A(\mathbf{x}) = A^{(0)} + A^{(1)}(\mathbf{x}), \quad A^{(0)} = \langle A(\mathbf{x}) \rangle \tag{1.4}$$

$$\mathbf{x} = \{x_1;\ x_2;\ x_3\}, \quad A = u_i,\ \varepsilon_{ij},\ \sigma_{ij}$$

The superscript 1 in (1.3) and (1.4) is attributed to fluctuations in the appropriate quantities.

Substituting (1.2) into the equilibrium equations and using the representation of the quantities in (1.2) in the form (1.3) and (1.4), we obtain a system of equations for the first approximation

$$\begin{aligned} &\theta_{,i}^{(1)} + a_1 u_{i,kk}^{(1)} + a_2 u_{i,33}^{(1)} + a_3 u_{3,i3} = -f_i \\ &\theta_{,3}^{(1)} + a_4 u_{3,kk}^{(1)} + a_5 u_{3,33}^{(1)} = -f_3 \end{aligned} \tag{1.5}$$

$$\begin{aligned} &a_1 = \mu^{(0)}/B_1,\ a_2 = \rho^{(0)}/B_1,\ a_3 = (\varkappa^{(0)} + \rho^{(0)})/B_1 \\ &a_4 = (\mu^{(0)} + \rho^{(0)})/B_2,\quad a_5 = (\varkappa^{(0)} + \gamma^{(0)} + 2\rho^{(0)})/B_2 \\ &B_1 = \lambda^{(0)} + \mu^{(0)},\quad B_2 = \lambda^{(0)} + \mu^{(0)} + \varkappa^{(0)} + \rho^{(0)} \\ &f_i = \frac{1}{B_1}\left(\theta^{(0)}\lambda_{,i}^{(1)} + 2\varepsilon_{ij}^{(0)}\mu_{,j}^{(1)} + \varepsilon_{33}^{(0)}\varkappa_{,i}^{(1)} + 2\varepsilon_{i3}^{(0)}\rho_{,3}^{(1)}\right) \\ &f_3 = \frac{1}{B_2}\left[\theta^{(0)}(\lambda^{(1)} + \varkappa^{(1)})_{,3} + 2\varepsilon_{ij}^{(0)}(\mu^{(1)} + \rho^{(1)})_{,j} + \right. \\ &\left.\quad \varepsilon_{33}^{(0)}(\varkappa^{(1)} + \gamma^{(1)} + 2\rho^{(1)})_{,3}\right] \quad (i = 1, 2;\ j, k = 1, 2, 3) \end{aligned} \tag{1.6}$$

Let us limit ourselves to the consideration of the case when the random fields $q^{(1)}(\mathbf{x}_*)$ are statistically homogeneous and isotropic and are statistically homogeneously and isotropically interrelated. In this case $q^{(1)}(\mathbf{x}_*)$ are representable /3/ by Fourier–Stieltjes integrals

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$$q^{(1)}(\mathbf{x}_*) = \iint\limits_{-\infty}^{\infty} \exp(i\boldsymbol{\omega}\mathbf{x}_*)\, d\varphi_q(\boldsymbol{\omega}); \qquad \boldsymbol{\omega} = \{\omega_1; \omega_2\} \tag{1.7}$$

We seek the solution of the system (1.5) in the form

$$u_j^{(1)} = v_j^{(1)} + w_j^{(1)} \quad (j = 1,\ 2,\ 3) \tag{1.8}$$

($v_j^{(1)}$ is a particular solution of (1.5), and $w_j^{(1)}$ is the general solution of a homogeneous system of equations corresponding to (1.5)). Let us put

$$v_j^{(1)} = \iint\limits_{-\infty}^{\infty} \alpha_q^{(j)}(\boldsymbol{\omega}) \exp(i\boldsymbol{\omega}\mathbf{x}_*)\, d\varphi_q(\boldsymbol{\omega}) \quad (j = 1, 2, 3) \tag{1.9}$$

(Here and henceforth, the summation is over the repeated subscript q, where q takes on the letter values mentioned earlier). Substituting (1.7) and (1.9) into (1.5), we obtain

$$\alpha_q^{(i)} = -\frac{b_i^{(q)}(1+a_1)\,\omega^2 - \omega_i b_l^{(q)}\omega_l}{a_1(1+a_1)\,\omega^4}$$
$$\alpha_q^{(3)} = -\frac{b_3^{(q)}}{a_4\omega^2} \quad (i, l = 1, 2), \quad \omega^2 = \omega_k\omega_k \quad (k = 1, 2) \tag{1.10}$$

where the $b_k^{(q)}$ are coefficients for the corresponding $d\varphi_q(\omega)$ in (1.6).

The general solution of the homogeneous system of equations corresponding to (1.5) has the form

$$w_j^{(1)} = \iint\limits_{-\infty}^{\infty} P_j^{(q)}(x_3) \exp(i\boldsymbol{\omega}\mathbf{x}_*)\, d\varphi_q(\boldsymbol{\omega}) \quad (j = 1, 2, 3) \tag{1.11}$$
$$P_r^{(q)} = (-1)^{r-1}\frac{\omega_1}{\omega_r} A_1^{(rq)} \exp(k_1\omega x_3) + \frac{\omega_r}{\omega} A_l^{(rq)} \exp(k_l\omega x_3)$$
$$P_3^{(q)} = A_l^{(3q)} \exp(k\,\omega x_3) \quad (r = 1, 2;\ l = 2, 3)$$

Here k_i $(i = 1, 2, 3)$ are roots of the characteristic equation

$$h_1k^6 + h_2\omega^2k^4 + h_3\omega^4k^2 + h_4\omega^6 = 0$$
$$h_n = h_n(a_1, a_2, a_3, a_4, a_5),\ n = 1, 2, 3, 4$$

that satisfy the condition $\operatorname{Re} k_s < 0$ $(s = 1, 2, \ldots, 6)$ (we later denote the roots satisfying this condition by $k_i = -m_i\omega,\ m_i > 0$), and $A_l^{(rq)}$ are arbitrary constants (there is no summation over r).

The boundary conditions for the stress fluctuations on the plane $x_3 = 0$ in the case of an arbitrary macro-homogeneous state of stress and strain have the form

$$\sigma_{13}^{(1)} = \sigma_{23}^{(1)} = \sigma_{33}^{(1)} = 0 \tag{1.12}$$

The solution of the system (1.5) can be obtained by substituting (1.9) and (1.11) into (1.8) and by determining the constants $A_l^{(rq)}$ from the conditions (1.12). The general expressions for the dispalcement fluctuations in the case of an arbitrary macro-homogeneous state of stress and strain are not presented here because of their awkwardness.

2. We perform all the subsequent calculations for two relatively simple, but nevertheless representative, particular cases: a) tension of a half-space in the direction of the axes x_1 and x_2 by constant stresses $\langle\sigma_{11}\rangle = \langle\sigma_{22}\rangle = \sigma^{(0)}$, and b) deformation of a half-space by constant tangential stresses $\langle\sigma_{13}\rangle = \langle\sigma_{23}\rangle = \sigma^{(0)}$.

The solutions of (1.5) will have the form

For the case a)

$$u_j^{(1)} = \iint\limits_{-\infty}^{\infty} \Big[\alpha_q^{(j)}(\boldsymbol{\omega}) + (-1)^{j-1}\frac{\omega_1}{\omega_j} A_1^{(jq)} \exp(-m_1\omega x_3) + \tag{2.1}$$
$$\frac{\omega_j}{\omega} A_l^{(jq)} \exp(-m_l\omega x_3)\Big] \exp(i\boldsymbol{\omega}\mathbf{x}_*)\, d\varphi_q(\boldsymbol{\omega})$$
$$u_3^{(1)} = \iint\limits_{-\infty}^{\infty} A_l^{(3q)} \exp(-m_l\omega x_3) \exp(i\boldsymbol{\omega}\mathbf{x}_*)\, d\varphi_q(\boldsymbol{\omega})$$

For the case b)

$$u_j^{(1)} = \iint_{-\infty}^{\infty} \Big[(-1)^{j-1} \frac{\omega_1}{\omega_j} A_1^{(jq)} \exp(-m_1 \omega x_3) + \frac{\omega_j}{\omega} A_l^{(jq)} \exp(-m_l \omega x_3) \Big] \exp(i \boldsymbol{\omega} \mathbf{x}_*)\, d\varphi_q(\boldsymbol{\omega}) \tag{2.2}$$

$$u_3^{(1)} = \iint_{-\infty}^{\infty} [\alpha_q^{(3)}(\boldsymbol{\omega}) + A_l^{(3q)} \exp(-m_l \omega x_3)] \exp(i \boldsymbol{\omega} \mathbf{x}_*)\, d\varphi_q(\boldsymbol{\omega})$$

$$(j = 1, 2;\ l = 2, 3)$$

(there is no summation over the j). The coefficients $A_l^{(mn)}$ are determined from the conditions (1.12) and are not presented here.

Using the Cauchy formula $\varepsilon_{ij}^{(1)} = (u_{i,j}^{(1)} + u_{j,i}^{(1)})/2$, the Hooke's law (1.2), and expressions for the displacements (2.1) and (2.2), the strains and stresses can be found in a first approximation. Here we limit ourselves to the strains. For case a) we have

$$\varepsilon_{km}^{(1)} = \frac{i}{2} \iint_{-\infty}^{\infty} \Big\{ \Big[\alpha_q^{(k)}(\boldsymbol{\omega}) + (-1)^{k-1} \frac{\omega_1}{\omega_k} A_1^{(kq)} \exp(-m_1 \omega x_3) + \frac{\omega_k}{\omega} A_l^{(kq)} \exp(-m_l \omega x_3) \Big] \omega_m + \Big[\alpha_q^{(m)}(\boldsymbol{\omega}) + (-1)^{m-1} \frac{\omega_1}{\omega_m} A_1^{(mq)} \times \exp(-m_1 \omega x_3) + \frac{\omega_m}{\omega} A_l^{(mq)} \exp(-m_l \omega x_3) \Big] \omega_k \Big\} \exp(i \boldsymbol{\omega} \mathbf{x}_*)\, d\varphi_q(\boldsymbol{\omega}) \tag{2.3}$$

$$(k, m = 1, 2;\ l = 2, 3)$$

(summation is not performed over the k and m). The expressions for the remaining strain fluctuations in case a), as well as for the strain fluctuations in case b) have an analogous structure, and are not presented here.

Let the correlation functions of the fields of elastic characteristics

$$K_{yz}(\boldsymbol{\xi}_*) = K_{yz}(\xi_*) = \langle \overline{y^{(1)}(\mathbf{x}_*)}\, z^{(1)}(\mathbf{x}_* + \boldsymbol{\xi}_*) \rangle$$

$$\xi_*^2 = \xi_m \xi_m\ (m = 1, 2),\ \ y, z = \lambda, \mu, \varkappa, \gamma, \rho$$

(here and henceforth the bar denotes the complex-conjugate quantity) satisfy the condition

$$\int_0^{\infty} |K_{yz}(\xi_*)|\, d\xi_* < \infty$$

In this case the following relationship holds for $d\varphi_q$

$$\langle \overline{d\varphi_y(\boldsymbol{\omega})}\, d\varphi_z(\boldsymbol{\omega}') \rangle = S_{yz}(\omega)\, \delta(\omega_1 - \omega_1')\, \delta(\omega_2 - \omega_2')\, d\boldsymbol{\omega}\, d\boldsymbol{\omega}' \tag{2.4}$$

($S(\omega)$ is the spectral density, and $\delta(t)$ is the delta function).

Using (2.3) for the strain fluctuations and (2.4), the correlation strain tensors can be constructed

$$K_{mnst}(\boldsymbol{\xi}_*, x_3, \xi_3) = \langle \overline{\varepsilon_{mn}^{(1)}(\mathbf{x}_*, x_3)}\, \varepsilon_{st}^{(1)}(\mathbf{x}_* + \boldsymbol{\xi}_*, x_3 + \xi_3) \rangle \tag{2.5}$$

for cases a) and b). Thus, for example, we have for the case a)

$$K_{mnst} = \frac{1}{4} \iint_{-\infty}^{\infty} \Big\{ \Big[\overline{\alpha_y^{(m)}} + (-1)^{m-1} \frac{\omega_1}{\omega_m} \overline{A_1^{(my)}} \exp(-m_1 \omega x_3) + \frac{\omega_m}{|\omega} \overline{A_l^{(my)}} \exp(-m_l \omega x_3) \Big] \Big[\alpha_z^{(s)} + (-1)^{s-1} \frac{\omega_1}{\omega_s} A_1^{(sz)} \times \exp(-m_1 \omega (x_3 + \xi_3)) + \frac{\omega_s}{\omega} A_k^{(sz)} \exp(-m_k \omega (x_3 + \xi_3)) \Big] \omega_n \omega_t + \Big[\overline{\alpha_y^{(m)}} + (-1)^{m-1} \frac{\omega_1}{\omega_m} \overline{A_1^{(my)}} \exp(-m_1 \omega x_3) + \frac{\omega_m}{\omega} \overline{A_l^{(my)}} \exp(-m_l \omega x_3) \Big] \Big[\alpha_z^{(t)} + (-1)^{t-1} \frac{\omega_1}{\omega_t} A_1^{(tz)} \times \exp(-m_1 \omega (x_3 + \xi_3)) + \frac{\omega_t}{\omega} A_k^{(tz)} \exp(-m_k \omega (x_3 + \xi_3)) \Big] \omega_n \omega_s + \Big[\overline{\alpha_y^{(n)}} + (-1)^{n-1} \frac{\omega_1}{\omega_n} \overline{A_1^{(ny)}} \exp(-m_1 \omega x_3) + \frac{\omega_n}{\omega} \overline{A_l^{(ny)}} \times \exp(-m_l \omega x_3) \Big] \Big[\alpha_z^{(s)} + (-1)^{s-1} \frac{\omega_1}{\omega_s} A_1^{(sz)} \exp(-m_1 \omega (x_3 + \xi_3)) + \tag{2.6}$$

$$\frac{\omega_s}{\omega} A_k^{(sz)} \exp(-m_k\omega(x_3+\xi_3))\Big]\,\omega_m\omega_t +$$
$$\Big[\overline{\alpha_y^{(n)}} + (-1)^{n-1}\frac{\omega_1}{\omega_n}\overline{A_1^{(ny)}}\exp(-m_1\omega x_3) + \frac{\omega_n}{\omega}\overline{A_l^{(ny)}} \times$$
$$\exp(-m_l\omega x_3)\Big]\Big[\alpha_z^{(t)} + (-1)^{t-1}\frac{\omega_1}{\omega_t}A_1^{(tz)}\exp(-m_1\omega(x_3+\xi_3)) +$$
$$\frac{\omega_t}{\omega}A_k^{(tz)}\exp(-m_k\omega(x_3+\xi_3))\Big]\omega_s\omega_m\Big\} S_{yz}(\omega)\exp(i\omega\xi_*)\,d\omega$$
$$(m,\ n,\ s,\ t = 1,\ 2;\ l,\ k = 2,\ 3;\ y,\ z = \lambda,\ \mu,\ \varkappa,\ \gamma,\ \rho)$$

(summation is performed over the subscripts y and z, where y and z take on the letter values mentioned; there is no summation over the m, n, s, t). The expressions for the remaining correlation strain tensor components are not presented here for the case a), nor for the case b).

3. We examine the variance of the strain in greater detail, where their expressions can be obtained for both case a) and case b) from the relationship (2.6) by setting $\xi_1 = \xi_2 = \xi_3 = 0$. Because of the awkwardness of the general expressions for the variance, we shall not present them here. Simpler formulas, and more easily subjected to analysis, are obtained for the mentioned random strain field characteristics in the case of considering some specific material. It is known /4/ that above 47 rocks can be considered as transversally isotropic media. Let us examine one, namely, marble,

It is convenient for the subsequent investigation to go from the elastic moduli $\lambda, \mu, \varkappa, \gamma, \rho$ to the engineering moduli /1/ E, E', G, G', ν, ν' between which the connection is given by the relationships

$$\begin{aligned}
&\lambda = E\,(E'\nu + E\nu'^2)\,F^{-1}, \quad \mu = G \\
&\varkappa = E\,[E'\nu'\,(1+\nu) - E'\nu - E\nu'^2]\,F^{-1} \\
&\gamma = [E'^2\,(1-\nu^2) + EE'\nu + E^2\nu'^2 - 2EE'\nu'\,(1+\nu)]\,F^{-1} - \\
&\quad 4G' + 2G, \quad \rho = G' - G, \quad F = (1+\nu)(E'\nu + E\nu'^2)
\end{aligned} \tag{3.1}$$

Following /4/, we introduce the anisotropy parameter p by the relationship $p = E/E'$. Moreover, we use the approximate formula /4/ which relates the shear modulus G' for planes normal to the plane of isotropy to the principal Young's moduli E, E' and the Poisson ratios ν, ν'

$$G' = \frac{EE'}{E + E'\,(1+2\nu')} \tag{3.2}$$

as well as by the known relationship that holds in the plane of isotropy

$$G = \frac{E}{2\,(1+\nu)} \tag{3.3}$$

Furthermore, assuming that the quantitites ν, ν', p are known (for marble $\nu = 0.22$, $\nu' = 0{,}06$; $p = 1{,}37$), while the representation (1.3) of $E(\mathbf{x}_*) = E^{(0)} + E^{(1)}(\mathbf{x}_*)$ or $E(\mathbf{x}_*)/E^{(0)} = 1 + E^{(1)}(\mathbf{x}_*)/E^{(0)}$ holds for the principal modulus E, we obtain expressions for the variance of the strain from (2.6):

Case a)

$$\begin{aligned}
D_{1212} = c_0 \int\!\!\int_{-\infty}^{\infty} &[c_1^{(1)} + c_2^{(1)}\exp(-2m_2\omega x_3) + \\
&c_3^{(1)}\exp(-(m_2+m_3)\,\omega x_3) + c_4^{(1)}\exp(-2m_3\omega x_3) + \\
&c_5^{(1)}\exp(-m_2\omega x_3) + c_6^{(1)}\exp(-m_3\omega x_3)]\,S(\omega)\,\frac{\omega_1^2\omega_2^2}{\omega^4}\,d\omega
\end{aligned} \tag{3.4}$$

$$D_{1313} = c_0 \int\!\!\int_{-\infty}^{\infty} c_7^{(1)}\,[\exp(-m_2\omega x_3) - \exp(-m_3\omega x_3)]^2\,S(\omega)\,\frac{\omega_1^2}{\omega^2}\,d\omega$$

$$D_\theta = c_0 \int\!\!\int_{-\infty}^{\infty} [c_8^{(1)} + c_9^{(1)}\exp(-m_2\omega x_3) + c_{10}^{(1)}\exp(-m_3\omega x_3)]^2\,S(\omega)\,d\omega$$

Case b)

$$D_{1212} = c_0 \int_{-\infty}^{\infty}\int \{c_1^{(2)} (\omega_1 - \omega_2)^4 (\omega_1 + \omega_2)^2 \exp(-2m_1\omega x_3) + \tag{3.5}$$
$$\omega_1^2\omega_2^2 (\omega_1 + \omega_2)^2 [c_2^{(2)} \exp(-2m_2\omega x_3) + c_3^{(2)} \exp(-2m_3\omega x_3) +$$
$$c_4^{(2)} \exp(-(m_2 + m_3)\omega x_3)] - \omega_1\omega_2 (\omega_1^2 - \omega_2^2) \times$$
$$[c_5^{(2)} \exp(-(m_1 + m_2)\omega x_3) + c_6^{(2)} \exp(-(m_1 + m_3)\omega x_3)]\} \frac{S(\omega)}{\omega^6}\, d\omega$$

$$D_{1313} = c_0 \int_{-\infty}^{\infty}\int c_7^{(2)} \{-\omega_1(\omega_1 + \omega_2) + \omega_2(\omega_2 - \omega_1) \exp(-m_1\omega x_3) +$$
$$\omega_1(\omega_1 + \omega_2) [c_8^{(2)} \exp(-m_2\omega x_3) + c_9^{(2)} \exp(-m_3\omega x_3)]\} \frac{S(\omega)}{\omega^4}\, d\omega$$

$$D_\theta = c_0 \int_{-\infty}^{\infty}\int [c_{10}^{(2)} \exp(-m_2\omega x_3) + c_{11}^{(2)} \exp(-m_3\omega x_3)]^2 S(\omega) \frac{(\omega_1 + \omega_2)^2}{\omega^2}\, d\omega$$

($S(\omega)$ is the spectral density of the variance of the modulus $E/E^{(0)}$, $c_0 = (\sigma^{(0)}/E^{(0)})^2$, $c_i^{(j)}$ are certain numbers).

We assume that the correlation function of the modulus $E/E^{(0)}$ has the form

$$K_E = d^2 e^{-\beta|\xi|} \tag{3.6}$$

Here $\beta > 0$ is a quantity reciprocal to the radius of correlation of the modulus $E/E^{(0)}$, and d^2 is the variance of the modulus $(d^2 < 1)$.

A spectral density of the form

$$S(\omega) = d^2 \frac{\beta\omega}{2\pi(\beta^2 + \omega^2)^{3/2}} \tag{3.7}$$

corresponds /3/ to the correlation function (3.6).

Substituting (3.7) into (3.4), and (3.5), passing to polar coordinates in these formulas, and integrating, we obtain

Case a)

$$D_{1212}^* = b_1^{(1)} - x\,[b_2^{(1)}\Delta(2m_2x) - b_3^{(1)}\Delta((m_2 + m_3)x) +$$
$$b_4^{(1)}\Delta(2m_3x) + b_5^{(1)}\Delta(m_2x) - b_6^{(1)}\Delta(m_3x)]$$

Case b)

$$D_{1212}^* = b_1^{(2)} - x\,[b_2^{(2)}\Delta(2m_1x) + b_3^{(2)}\Delta(2m_2x) + b_4^{(2)}\Delta(2m_3x) - b_5^{(2)}\Delta((m_2 + m_3)x)]$$
$$(D_{1212}^* = D_{1212}/(c_0d^2),\ \Delta(z) = H_0(z) - N_0(z),\ x = \beta x_3)$$

Here $H_0(z)$, $N_0(z)$ are Struve and Neumann functions, respectively, and $b_i^{(j)}$ are numbers. Expressions for the remaining variances are analogous in structure and are not presented here.

The solid lines in the Fig.1 are graphs of the variance of the strain ($1 - D_{1212}^*\cdot 10$, $2 - D_\theta^*$, $3 - D_{1313}^*\cdot 10^3$) for the case a) and ($1' - D_{1212}^*$, $2 - D_\theta^*\cdot 10^{-1}$, $3 - D_{1313}^*$) for case b). It is seen that the domain in which changes in the variance are essential does not exceed 3–7 radii of correlation of the elastic modulus, while the greatest values of the variance are either on the boundary $x = 0$ or in the zone of the size 1–2 of the radius of correlation of the modulus. The corresponding variance curves for the strain of a micro-inhomogeneous half-space with "averaged" (isotropic half-space) /5/ elastic moduli are presented by dashed lines for comparison. The solution of the problem for an isotropic half-space is presented in /6/.

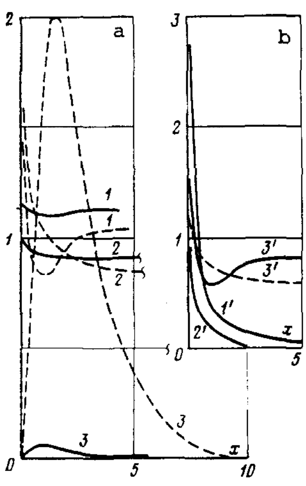

Fig.1

It should be noted that for shear (case b) the bulk expansion θ is zero /1/ in a transversally-isotropic homogeneous medium. It is different from zero for a micro-inhomogeneous transversally-isotropic medium, has a maximal variance on the boundary of the half-space $x = 0$ which damps out rapidly as x grows (Fig. 1b). The bulk expansion θ and shear ε_{12} are zero in a micro-inhomogeneous isotropic medium as in a homogeneous medium.

By calculating the coefficients of variability v (the ratio between the root-mean-square deviations of the strain fluctuations at the boundary to the root-mean-square deviations at infinity)

/7/, we obtain: anisotropic medium a) $v_{1212} = 1.015$; $v_\theta = 1.049$; b) $v_{1313} = 1.414$; isotropic medium a) $v_{1212} = 1.192$; $v_\theta = 1.404$; b) $v_{1313} = 1.414$. Comparing these results, it can be noted that the coefficients of variability for a transversally-isotropic medium do not exceed the corresponding coefficients for an isotropic medium. This comparison permits the assumption that a transversally-isotropic medium is less "contrasty" than an isotropic medium. This phenomenon, as well as the presence of the volume expansion in the case of shear, should be taken into account in solving practical problems.

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